

# One-loop Kähler potential in non-renormalizable theories

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## Abstract

We consider a general  $d=4$   $N=1$  globally supersymmetric lagrangian involving chiral and vector superfields, with arbitrary superpotential, Kähler potential and gauge kinetic function. We compute perturbative quantum corrections by employing a component field approach that respects supersymmetry and background gauge invariance. In particular, we obtain the full one-loop correction to the Kähler potential in supersymmetric Landau gauge. Two derivations of this result are described. The non-renormalization of the superpotential and the quadratic correction to the Fayet-Iliopoulos terms are further checks of our computations.

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# 1 Introduction

Perturbative quantum corrections have a peculiar form in supersymmetric theories. In the case of  $d=4$   $N=1$  theories, in particular, the non-renormalization theorem [1] establishes that the superpotential remains uncorrected, whereas the Kähler potential generically receives quantum corrections. The anomalous dimensions of chiral superfields are the simplest example of the latter effects, but a richer structure emerges if the full field dependence of such corrections is taken into account. Full one-loop corrections to the Kähler potential have been recently computed, both in the Wess-Zumino model [2] and in more general renormalizable models [3, 4, 5]. For the most general renormalizable  $N=1$  theory, the one-loop correction to the Kähler potential was found to have a very compact form in supersymmetric Landau gauge. The result reads [5]:

$$\Delta K = -\frac{1}{32\pi^2} \left[ \text{Tr} \left( \mathcal{M}_\phi^2 \left( \log \frac{\mathcal{M}_\phi^2}{\Lambda^2} - 1 \right) \right) - 2 \text{Tr} \left( \mathcal{M}_V^2 \left( \log \frac{\mathcal{M}_V^2}{\Lambda^2} - 1 \right) \right) \right] \quad (1)$$

where  $\mathcal{M}_\phi^2$  and  $\mathcal{M}_V^2$  are the (chiral superfield dependent) mass matrices in the chiral and vector superfield sectors, respectively, and  $\Lambda$  is an ultraviolet cutoff. One-loop corrections to the Kähler potential have also been investigated in non-renormalizable  $N=1$  models, divergent contributions being the main focus. For instance, quadratically divergent corrections to the Kähler potential in general models were computed in [6]. Quadratic and logarithmic divergences were also studied in general supergravity models with diagonal gauge kinetic function [7], or in models with chiral superfields only [8]. In [9], the Wilsonian evolution of the Kähler potential was studied in non-renormalizable models with an abelian vector superfield and/or gauge singlet chiral superfields. Divergent and finite corrections in specific models were also evaluated in [10].

The main purpose of this paper is to generalize the result (1) to non-renormalizable theories, i.e. to compute the full (divergent and finite) one-loop correction to the Kähler potential in a general globally supersymmetric theory. Our perturbative calculation starts from a tree-level lagrangian in which the superpotential, the Kähler potential, the gauge kinetic function, the gauge group and the matter representations are arbitrary. Upon quantizing the theory, a supersymmetric gauge fixing term is added, and we choose to preserve supersymmetric background gauge invariance. This framework is then translated to the component field level and quantum corrections are computed in terms of component Feynman diagrams. Notice that we do *not* choose the Wess-Zumino gauge, supplemented by a gauge fixing term for the component vector fields. Instead, we keep all components of quantum supermultiplets and use supersymmetric Landau gauge. Thus our component computations are equivalent to superfield computations, and a superfield language can be used to interpret our results. In particular, we obtain the full one-loop correction to the Kähler potential, which is our main result. In spite of the fact that the interactions are considerably more complicated in comparison to the renormalizable case, we find that the logarithmically divergent and finite one-loop corrections to the Kähler potential can be cast in the same form as in eq. (1), with generalized mass matrices  $\mathcal{M}_\phi^2$  and  $\mathcal{M}_V^2$ . In addition to that, the Kähler potential receives a quadratically divergent correction, in agreement with [6]. A consistency check based on supersymmetric background gauge invariance is also discussed. The non-renormalization of the superpotential and the quadratic correction

to the Fayet-Iliopoulos terms are further checks of our computations.

## 2 Theoretical framework

We consider a general  $d=4$   $N=1$  globally supersymmetric theory defined by a tree-level lagrangian of the form (see e.g. [11, 12]):

$$\mathcal{L} = \left[ \int d^2\theta w(\hat{\phi}) + \text{h.c.} \right] + \int d^4\theta \left[ K(\hat{\phi}, e^{2\hat{V}}\hat{\phi}) + 2\kappa_a \hat{V}^a \right] + \left[ \int d^2\theta \frac{1}{4} f_{ab}(\hat{\phi}) \hat{\mathcal{W}}^a \hat{\mathcal{W}}^b + \text{h.c.} \right]. \quad (2)$$

The theory has a general (possibly product) gauge group  $G$ , with hermitian generators  $T_a$  satisfying the Lie algebra  $[T_a, T_b] = ic_{ab}{}^c T_c$ . The associated vector superfields  $\hat{V} = \hat{V}^a T_a$  have superfield strengths  $\hat{\mathcal{W}} = \hat{\mathcal{W}}^a T_a = -\frac{1}{8} \bar{D}_{\dot{\alpha}} \bar{D}^{\dot{\alpha}} e^{-2\hat{V}} D_{\alpha} e^{2\hat{V}}$ . The chiral superfields  $\hat{\phi} = \{\hat{\phi}^i\}$  belong to a general (reducible) representation of  $G$ . Supersymmetric gauge transformations read  $e^{2\hat{V}} \rightarrow e^{-2i\hat{\Lambda}^\dagger} e^{2\hat{V}} e^{2i\hat{\Lambda}}$  and  $\hat{\phi} \rightarrow e^{-2i\hat{\Lambda}} \hat{\phi}$ , where  $\hat{\Lambda} = \hat{\Lambda}^a T_a$  is chiral. The Fayet-Iliopoulos coefficients  $\kappa_a$  are real and may be nonvanishing only for the abelian factors of the gauge group  $G$ . The superpotential  $w$ , the Kähler potential  $K$  and the gauge kinetic function  $f_{ab}$  are only constrained by gauge invariance, and are otherwise arbitrary. In more detail:  $w(\hat{\phi})$  is  $G$ -invariant,  $K(\hat{\phi}, \hat{\phi})$  is real and  $G$ -invariant, and  $f_{ab}(\hat{\phi})$  transforms as a symmetric product of adjoint representations of  $G$ . These constraints are expressed by the identities:

$$w_i(\hat{\phi})(T_a \hat{\phi})^i \equiv 0 \quad (3)$$

$$K_i(\hat{\phi}, \hat{\phi})(T_a \hat{\phi})^i \equiv (\hat{\phi} T_a)^{\bar{i}} K_{\bar{i}}(\hat{\phi}, \hat{\phi}) \quad (4)$$

$$f_{abi}(\hat{\phi})(T_c \hat{\phi})^i \equiv ic_{ac}{}^d f_{db}(\hat{\phi}) + ic_{bc}{}^d f_{ad}(\hat{\phi}) \quad (5)$$

where  $(T_a \hat{\phi})^i \equiv (T_a)^i{}_j \hat{\phi}^j$ ,  $(\hat{\phi} T_a)^{\bar{i}} \equiv \hat{\phi}^{\bar{j}} (T_a)^{\bar{i}}{}_{\bar{j}}$ ,  $w_i(\hat{\phi}) \equiv \partial w(\hat{\phi}) / \partial \hat{\phi}^i$ , and so on. Further identities can be obtained by differentiating the ones above. We also recall that, in the special case of a renormalizable theory,  $w$  is at most cubic in the chiral superfields,  $K$  is quadratic (i.e. canonical) and  $f_{ab}$  is constant (i.e. canonical). Here we aim at full generality and do not impose renormalizability.

The supersymmetric lagrangian above is the most general one that contains no more than two space-time derivatives on component fields<sup>1</sup>. It could arise as a low-energy limit of a more fundamental theory. In fact, since we take  $w, K, f_{ab}$  to be generic functions, not restricted by renormalizability, the lagrangian  $\mathcal{L}$  in eq. (2) necessarily describes an effective theory, valid below some cutoff scale  $\Lambda$ . For consistency reasons,  $\Lambda$  cannot be much larger than the scale whose inverse powers control the non-renormalizable terms in  $w, K, f_{ab}$ . Anyhow, we will not address the origin of the lagrangian itself. We will just take  $\mathcal{L}$  as a general classical bare lagrangian and study the corresponding one-loop corrections. In principle, such self-corrections could be matched to those of the hypothetical underlying theory, if the latter theory were known.

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<sup>1</sup>This is already a non-trivial generalization of the renormalizable case. A further generalization would be the inclusion of supersymmetric higher derivative terms, which are additional non-renormalizable terms. We leave this to future investigation.

Quantum corrections to a classical supersymmetric lagrangian can be computed by various techniques. We find it convenient to use the background field method [1, 12]. The superfields  $\hat{\phi}, \hat{V}$  are split into background (still denoted by  $\hat{\phi}, \hat{V}$ ) and quantum ( $\phi, V$ ) parts. A supersymmetric gauge fixing lagrangian  $\mathcal{L}_{\text{gf}}$  is added in order to break the quantum gauge invariance, and a corresponding ghost lagrangian  $\mathcal{L}_{\text{gh}}$  is introduced. If we choose to preserve supersymmetric background gauge invariance, we can take [1, 12]:

$$\hat{\phi} \longrightarrow \hat{\phi} + \phi, \quad (6)$$

$$e^{2\hat{V}} \longrightarrow e^{\hat{V}} e^{2V} e^{\hat{V}}, \quad (7)$$

$$\mathcal{L}_{\text{gf}} = -\frac{1}{8\xi} \int d^4\theta (\nabla^2 V)^a (\bar{\nabla}^2 V)^a, \quad (8)$$

where the background vector superfield has been put in a convenient form,  $\xi$  is a gauge parameter and  $\nabla_\alpha \equiv e^{-\hat{V}} D_\alpha e^{\hat{V}}$ ,  $\bar{\nabla}_{\dot{\alpha}} \equiv e^{\hat{V}} \bar{D}_{\dot{\alpha}} e^{-\hat{V}}$  are background gauge covariant supersymmetric derivatives. Ghost superfields interact with vector superfields only, and we will not need the explicit expression of  $\mathcal{L}_{\text{gh}}$ . In the abelian case, the splitting of the vector superfield in (7) reduces to  $\hat{V} \longrightarrow \hat{V} + V$  and the gauge fixing term (8) becomes

$$\mathcal{L}_{\text{gf}} = -\frac{1}{8\xi} \int d^4\theta D^2 V^a \bar{D}^2 V^a. \quad (9)$$

Making these simpler choices in the non-abelian case is certainly allowed, but does not lead to a background gauge invariant effective action. Since we find it useful to preserve the latter property, we proceed in the way explained above.

Once the replacements (6), (7) have been made in (2) and the gauge fixing and ghost terms have been added, the resulting lagrangian can be expanded in powers of the quantum superfields. The zero-th order part is just the original lagrangian (2) for the classical (background) superfields,  $\mathcal{L}(\hat{\phi}, \hat{V})$ . The terms linear in quantum superfields do not contribute to the (one-particle-irreducible) effective action and can be dropped. The part bilinear in quantum superfields,  $\mathcal{L}_{\text{bil}}(\phi, V, \dots; \hat{\phi}, \hat{V})$ , is the relevant one for the computation of one-loop quantum corrections. One has to integrate out the quantum superfields in the theory defined by  $\mathcal{L} + \mathcal{L}_{\text{bil}}$ , either diagrammatically or by direct functional methods. The result of this operation gives the one-loop-corrected effective action, a functional of background superfields only. If a (super) derivative expansion of the latter functional is performed, the lowest order terms can be interpreted as corrections to the basic functions in (2). In other words, one-loop corrections to the effective lagrangian will have the form:

$$\Delta\mathcal{L} = \left[ \int d^2\theta \Delta w(\hat{\phi}) + \text{h.c.} \right] + \int d^4\theta \left[ \Delta K(\hat{\phi}, e^{2\hat{V}} \hat{\phi}) + 2 \Delta\kappa_a \hat{V}^a \right] + \dots \quad (10)$$

where the dots stand for terms containing supercovariant derivatives ( $\hat{\mathcal{W}}\hat{\mathcal{W}}$  terms and higher derivative terms), which we will not compute. Our purpose is to compute the corrections  $\Delta w$ ,  $\Delta K$  and  $\Delta\kappa_a$ .

### 3 Component field approach

The expected form (10) of quantum corrections relies on the assumption that quantum superfields are integrated out in a supersymmetric way. This is automatic if the perturbative computations are performed at the superfield level. Here we choose to work with component fields, instead. Nevertheless, we retain the whole off-shell structure of quantum supermultiplets and literally translate the framework described above to the component level. In particular, instead of simplifying the structure of vector supermultiplets by using the Wess-Zumino gauge, supplemented by a gauge fixing term for the component vector fields only, we work with full supermultiplets and use the manifestly supersymmetric gauge fixing term (8). Thus integrating out component fields is literally equivalent to integrating out superfields, and supersymmetric background gauge invariance can also be preserved.

In order to fix the notation, we recall the component expansion of chiral and vector superfields<sup>2</sup>:

$$\phi^i = \varphi^i + \sqrt{2}\theta\psi^i + \theta\theta F^i - i\theta\sigma^\mu\bar{\theta}\partial_\mu\varphi^i - \frac{i}{\sqrt{2}}\theta\theta\bar{\theta}\bar{\sigma}^\mu\partial_\mu\psi^i - \frac{1}{4}\theta\theta\bar{\theta}\bar{\theta}\square\varphi^i \quad (11)$$

$$\begin{aligned} V^a = & C^a + i\theta\chi^a - i\bar{\theta}\bar{\chi}^a + \frac{i}{\sqrt{2}}\theta\theta G^a - \frac{i}{\sqrt{2}}\bar{\theta}\bar{\theta}\bar{G}^a + \theta\sigma^\mu\bar{\theta}A_\mu^a \\ & + i\theta\bar{\theta}\bar{\theta}\left(\bar{\chi}^a - \frac{i}{2}\bar{\sigma}^\mu\partial_\mu\chi^a\right) - i\bar{\theta}\bar{\theta}\theta\left(\chi^a - \frac{i}{2}\sigma^\mu\partial_\mu\bar{\chi}^a\right) + \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}\left(D^a - \frac{1}{2}\square C^a\right) \end{aligned} \quad (12)$$

where  $C^a, D^a$  are real scalar fields,  $\varphi^i, F^i, G^a$  are complex scalar fields,  $A_\mu^a$  are real vector fields, and  $\psi^i, \chi^a, \lambda^a$  are complex Weyl fields. The expansions above apply to the quantum superfields  $\phi^i$  and  $V^a$ . Similar component expansions hold for the background superfields  $\hat{\phi}^i$  and  $\hat{V}^a$ , and one can eventually obtain the full component expansion of  $\mathcal{L}_{\text{bil}}$ , to be used for the computation of  $\Delta\mathcal{L}$ . However, since supersymmetry constrains one-loop corrections to have the form (10), a convenient choice of the background superfields can simplify the computation of the functions  $\Delta K$  and  $\Delta w$ . For instance, in order to compute the function  $\Delta K$ , we could choose a background in which the only non-vanishing fields are the scalars  $\hat{\varphi}^i(x)$  contained in  $\hat{\phi}^i$ . Then eq. (10) predicts that the one-loop computation should produce terms of the form  $\Delta K_{\bar{i}j}(\hat{\varphi}, \hat{\varphi})\partial_\mu\hat{\varphi}^{\bar{i}}\partial^\mu\hat{\varphi}^j$ . Hence the one-loop correction  $\Delta K_{\bar{i}j}(\hat{\varphi}, \hat{\varphi})$  to the Kähler metric could be identified and the functional form of the one-loop-corrected Kähler potential could be reconstructed by integration. Alternative choices of the background can be even more convenient. In what follows, we will make this choice:

$$\hat{\phi}^i = \hat{\varphi}^i + \theta\theta\hat{F}^i, \quad (13)$$

$$\hat{V}^a = \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}\hat{D}^a, \quad (14)$$

where both the physical scalars  $\hat{\varphi}^i$  and the auxiliary fields  $\hat{F}^i, \hat{D}^a$  are taken to be constant (i.e. space-time independent). If we specialize (10) to this background, we infer that the

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<sup>2</sup>Our conventions are slightly different from those of ref. [11]. For instance, we use the space-time metric  $g_{\mu\nu} = \text{diag}(+1, -1, -1, -1)$ , the Pauli matrices  $\sigma^\mu = (1, \vec{\sigma})$ ,  $\bar{\sigma}^\mu = (1, -\vec{\sigma})$ , and the supersymmetric derivatives  $D_\alpha = \partial/\partial\theta^\alpha - i\sigma^\mu_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_\mu$ ,  $\bar{D}_{\dot{\alpha}} = -\partial/\partial\bar{\theta}^{\dot{\alpha}} + i\theta^\alpha\sigma^\mu_{\alpha\dot{\alpha}}\partial_\mu$ .

one-loop computation should generate the terms:

$$\Delta\mathcal{L} = [\Delta w_i(\hat{\varphi})\hat{F}^i + \text{h.c.}] + \Delta K_{\bar{i}j}(\hat{\varphi}, \hat{\varphi})\hat{F}^{\bar{i}}\hat{F}^j + [\Delta K_j(\hat{\varphi}, \hat{\varphi})(T_a\hat{\varphi})^j + \Delta\kappa_a]\hat{D}^a + \dots \quad (15)$$

The dots stand for higher order terms in  $\hat{F}$ ,  $\hat{D}$  and correspond to the (omitted)  $\hat{\mathcal{W}}\hat{\mathcal{W}}$  and higher derivative terms in (10). The computational advantage of using a background with constant  $\hat{\varphi}$ ,  $\hat{F}$ ,  $\hat{D}$  is obvious: the one-loop diagrams to be evaluated have vanishing external momenta. In other words, in such a background  $-\Delta\mathcal{L}$  is the one-loop correction to the effective potential  $\mathcal{V}_{\text{eff}}(\hat{\varphi}, \hat{F}, \hat{D})$ , considered as a function of both physical and auxiliary scalar fields. In the case of renormalizable models, one-loop computations of this object by either component or superfield techniques can be found, for instance, in refs. [13, 14]. Here we are interested in obtaining the first few terms in the auxiliary field expansion of the effective potential, in the general non-renormalizable theory defined above. Once the results of the one-loop computation have been cast in the form (15), the functions  $\Delta w$  and  $\Delta K$  can be reconstructed from their derivatives<sup>3</sup>  $\Delta w_i$ ,  $\Delta K_j$ ,  $\Delta K_{\bar{i}j}$ , and the coefficients  $\Delta\kappa_a$  can be easily identified, too. Notice that the one-loop correction to the Kähler potential can be reconstructed in two different ways, thanks to background gauge invariance: this allows us to make a non-trivial consistency check. In fact, we will proceed as follows. First, we will expand the lagrangian in a background with constant  $\hat{\varphi}$ ,  $\hat{F}$  and vanishing  $\hat{D}$  (section 4). Then we will compute the one-loop corrections to the terms linear and quadratic in  $\hat{F}$ , from which  $\Delta w$  and  $\Delta K$  can be reconstructed (sections 5 and 6). In the final part (section 7) we will instead consider a background with constant  $\hat{\varphi}$ ,  $\hat{D}$  and vanishing  $\hat{F}$ . Then we will compute the one-loop corrections to the terms linear in  $\hat{D}$ . This will allow us to identify  $\Delta\kappa_a$  and to derive  $\Delta K$  in a different way.

## 4 Quantum bilinears and propagators

We take the background chiral superfields in the form (13), with constant  $\hat{\varphi}$ ,  $\hat{F}$  fields, and plug the background-quantum splitting (6) in (2). At the same time, we take vanishing background vector superfields<sup>4</sup>. In this case, the background-quantum splitting (7) reduces to the simple replacement  $\hat{V} \rightarrow V$  in (2),  $\mathcal{L}_{\text{gf}}$  in (8) reduces to (9), and  $\mathcal{L}_{\text{gh}}$  is not relevant because ghosts do not interact with the (chiral) background. The lagrangian is then expanded and only the part bilinear in the quantum fields  $\phi, V$  is retained,  $\mathcal{L}_{\text{bil}} = \mathcal{L}_{\phi\phi} + \mathcal{L}_{VV} + \mathcal{L}_{\phi V}$ . Integration by parts is used whenever convenient, and some relations that follow from (4) are used to rearrange the terms generated by the expansion of the Kähler potential. The terms that make up  $\mathcal{L}_{\text{bil}}$  have a complicated dependence on  $\hat{\varphi}$  and a simple dependence (at most quadratic) on  $\hat{F}$ . Thus each of the three parts in  $\mathcal{L}_{\text{bil}}$  can in turn be decomposed as  $\mathcal{L}_{\phi\phi} = \mathcal{L}_{\phi\phi}^{(0)} + \mathcal{L}_{\phi\phi}^{(\hat{F})} + \mathcal{L}_{\phi\phi}^{(\hat{F}\hat{F})}$ ,  $\mathcal{L}_{VV} = \mathcal{L}_{VV}^{(0)} + \mathcal{L}_{VV}^{(\hat{F})} + \mathcal{L}_{VV}^{(\hat{F}\hat{F})}$ ,  $\mathcal{L}_{\phi V} = \mathcal{L}_{\phi V}^{(0)} + \mathcal{L}_{\phi V}^{(\hat{F})} + \mathcal{L}_{\phi V}^{(\hat{F}\hat{F})}$ , in a self-explanatory notation.

First of all we list the terms that do not depend on  $\hat{F}$ , i.e. the quantum bilinears in a

<sup>3</sup> Up to irrelevant constant terms in  $\Delta w$  or harmonic terms in  $\Delta K$ .

<sup>4</sup> Background gauge invariance will not be exploited directly till section 7.

pure constant  $\hat{\varphi}$  background:

$$\mathcal{L}_{\phi\phi}^{(0)} = \hat{K}_{\bar{i}j} \left( -\bar{\varphi}^{\bar{i}} \square \varphi^j + \bar{F}^{\bar{i}} F^j + i \bar{\psi}^{\bar{i}} \bar{\sigma}^{\mu} \partial_{\mu} \psi^j \right) + \left[ \hat{w}_{ij} \left( F^i \varphi^j - \frac{1}{2} \psi^i \psi^j \right) + \text{h.c.} \right] \quad (16)$$

$$\begin{aligned} \mathcal{L}_{VV}^{(0)} &= \hat{H}_{ab} \left[ -\frac{1}{4} (\partial_{\mu} A_{\nu}^a - \partial_{\nu} A_{\mu}^a) (\partial^{\mu} A^{\nu b} - \partial^{\nu} A^{\mu b}) + \frac{1}{2} D^a D^b + i \bar{\lambda}^a \bar{\sigma}^{\mu} \partial_{\mu} \lambda^b \right] \\ &+ \hat{S}_{ab} \left[ \frac{1}{2} A_{\mu}^a A^{\mu b} + C^a D^b - \frac{1}{2} C^a \square C^b + \bar{G}^a G^b - \left( \chi^a \lambda^b - \frac{i}{2} \chi^a \sigma^{\mu} \partial_{\mu} \bar{\chi}^b + \text{h.c.} \right) \right] \\ &- \frac{1}{\xi} \left[ \frac{1}{2} (\partial^{\mu} A_{\mu}^a)^2 + \frac{1}{2} (D^a - \square C^a)^2 + \partial^{\mu} \bar{G}^a \partial_{\mu} G^a + i (\bar{\lambda}^a + i \partial_{\rho} \chi^a \sigma^{\rho}) \bar{\sigma}^{\nu} \partial_{\nu} (\lambda^a - i \sigma^{\mu} \partial_{\mu} \bar{\chi}^a) \right] \end{aligned} \quad (17)$$

$$\mathcal{L}_{\phi V}^{(0)} = (\hat{\varphi} T_a)^{\bar{i}} \hat{K}_{\bar{i}j} \left[ \varphi^j (i \partial^{\mu} A_{\mu}^a + D^a - \square C^a) - i \sqrt{2} F^j \bar{G}^a + i \sqrt{2} \psi^j (\lambda^a - i \sigma^{\mu} \partial_{\mu} \bar{\chi}^a) \right] + \text{h.c.} \quad (18)$$

Most of the dependence on  $\hat{\varphi}$  is left implicit. In particular, we have used the abbreviations:

$$\hat{K}_{\bar{i}j} \equiv K_{\bar{i}j}(\hat{\varphi}, \hat{\varphi}), \quad \hat{w}_{ij} \equiv w_{ij}(\hat{\varphi}), \quad (19)$$

$$\hat{H}_{ab} \equiv \frac{1}{2} (f_{ab}(\hat{\varphi}) + \bar{f}_{ab}(\hat{\varphi})), \quad \hat{S}_{ab} \equiv (\hat{\varphi} T_a)^{\bar{i}} K_{\bar{i}j}(\hat{\varphi}, \hat{\varphi}) (T_b \hat{\varphi})^j + (\hat{\varphi} T_b)^{\bar{i}} K_{\bar{i}j}(\hat{\varphi}, \hat{\varphi}) (T_a \hat{\varphi})^j. \quad (20)$$

The matrices  $\hat{K}_{\bar{i}j}$  and  $\hat{w}_{ij}$  ( $\hat{H}_{ab}$  and  $\hat{S}_{ab}$ ) have the meaning of  $\hat{\varphi}$  dependent metric and masses in the  $\phi$  ( $V$ ) sector. Notice that  $\hat{K}_{\bar{i}j}$  is hermitian and  $\hat{w}_{ij}$  is symmetric, whereas  $\hat{H}_{ab}$  and  $\hat{S}_{ab}$  are real and symmetric. Next we list the terms that depend on  $\hat{F}$ :

$$\mathcal{L}_{\phi\phi}^{(\hat{F})} = \hat{F}^j \left[ \frac{1}{2} \hat{w}_{ikj} \varphi^i \varphi^k + \hat{K}_{\bar{i}kj} \bar{F}^{\bar{i}} \varphi^k + \hat{K}_{\bar{i}kj} \left( \bar{F}^{\bar{i}} \bar{\varphi}^{\bar{k}} - \frac{1}{2} \bar{\psi}^{\bar{i}} \bar{\psi}^{\bar{k}} \right) \right] + \text{h.c.} \quad (21)$$

$$\mathcal{L}_{VV}^{(\hat{F})} = \hat{F}^j \left[ -\frac{1}{2} (\hat{H}_{ab})_j \lambda^a \lambda^b + (\hat{S}_{ab})_j \left( \frac{1}{2} \bar{\chi}^a \bar{\chi}^b - i \sqrt{2} C^a \bar{G}^b \right) \right] + \text{h.c.} \quad (22)$$

$$\begin{aligned} \mathcal{L}_{\phi V}^{(\hat{F})} &= \hat{F}^j \left[ \left( \hat{K}_{\bar{i}k} (T_a)^k_j + \hat{K}_{\bar{i}kj} (T_a \hat{\varphi})^k \right) \left( 2 \bar{F}^{\bar{i}} C^a + i \sqrt{2} \bar{\psi}^{\bar{i}} \bar{\chi}^a - i \sqrt{2} \bar{\varphi}^{\bar{i}} \bar{G}^a \right) \right. \\ &\quad \left. - i \sqrt{2} (\hat{\varphi} T_a)^{\bar{i}} \hat{K}_{\bar{i}kj} \varphi^k \bar{G}^a \right] + \text{h.c.} \end{aligned} \quad (23)$$

$$\mathcal{L}_{\phi\phi}^{(\hat{F}\hat{F})} = \hat{F}^{\bar{i}} \hat{F}^j \left[ \hat{K}_{\bar{i}j\bar{k}\ell} \bar{\varphi}^{\bar{k}} \varphi^{\ell} + \frac{1}{2} \left( \hat{K}_{\bar{i}j\bar{k}\ell} \varphi^k \varphi^{\ell} + \hat{K}_{\bar{i}j\bar{k}\ell} \bar{\varphi}^{\bar{k}} \bar{\varphi}^{\bar{\ell}} \right) \right] \quad (24)$$

$$\mathcal{L}_{VV}^{(\hat{F}\hat{F})} = \hat{F}^{\bar{i}} \hat{F}^j (\hat{S}_{ab})_{\bar{i}j} C^a C^b \quad (25)$$

$$\mathcal{L}_{\phi V}^{(\hat{F}\hat{F})} = \hat{F}^{\bar{i}} \hat{F}^j \left[ 2 \left( (T_a)_{\bar{i}}^{\bar{\ell}} \hat{K}_{\bar{\ell}kj} + (\hat{\varphi} T_a)^{\bar{\ell}} \hat{K}_{\bar{\ell}kj} \right) \varphi^k C^a \right] + \text{h.c.} \quad (26)$$

Having completed the list of quantum bilinears in the chosen background, we could in principle compute the one-loop effective potential  $\mathcal{V}_{\text{eff}}(\hat{\varphi}, \hat{F}, \hat{D}=0)$ , which corresponds to diagrams with an arbitrary number of (zero momentum)  $\hat{\varphi}$  and  $\hat{F}$  external legs. In practice, as explained above, we are only interested in the first few terms in the  $\hat{F}$  expansion of the effective potential. This amounts to compute diagrams with an arbitrary number of  $\hat{\varphi}$  external legs and a small number of  $\hat{F}$  external legs. In order to take into account the full  $\hat{\varphi}$  dependence, we will proceed as follows. In the remainder of this section, we will consider the quantum bilinears in the pure constant  $\hat{\varphi}$  background and compute the  $\hat{\varphi}$ -dressed propagators for the quantum fields. In the next two sections, we will consider

the quantum bilinears that also depend on  $\hat{F}$  and treat them as  $\hat{\varphi}$ - and  $\hat{F}$ -dependent interaction vertices, to be joined by the  $\hat{\varphi}$ -dressed quantum propagators. In particular, we will compute the  $\hat{\varphi}$ -dressed one- and two-point functions of  $\hat{F}$ , which will allow us to reconstruct  $\Delta w$  and  $\Delta K$ , respectively.

The quantum propagators in a constant  $\hat{\varphi}$  background are obtained by inverting the quadratic forms that appear in (16), (17), (18). We omit the details of the derivation and write the results directly in momentum space. We use a compact matrix notation and denote by  $\hat{K}, \hat{w}, \hat{H}, \hat{S}$  the  $\hat{\varphi}$  dependent matrices defined in (19) and (20). The matrices  $\hat{K}$  and  $\hat{w}$  should not be confused with the functions  $K$  and  $w$  used to define them by double differentiation (the context should make this distinction clear, despite the slight abuse of notation).

From  $\mathcal{L}_{\phi\phi}^{(0)}$ , eq. (16), we obtain the  $\hat{\varphi}$ -dressed propagators for the components of quantum chiral superfields:

$$\langle \varphi^i \bar{\varphi}^{\bar{j}} \rangle_{\hat{\varphi}} = i \left[ \left( \hat{K} p^2 - \hat{w} \hat{K}^{-1 T} \hat{w} \right)^{-1} \right]^{i\bar{j}} \quad (27)$$

$$\langle F^i \bar{F}^{\bar{j}} \rangle_{\hat{\varphi}} = i p^2 \left[ \left( \hat{K} p^2 - \hat{w} \hat{K}^{-1 T} \hat{w} \right)^{-1} \right]^{i\bar{j}} \quad (28)$$

$$\langle F^i \varphi^j \rangle_{\hat{\varphi}} = -i \left[ \left( \hat{K} p^2 - \hat{w} \hat{K}^{-1 T} \hat{w} \right)^{-1} \hat{w} \hat{K}^{-1 T} \right]^{ij} \quad (29)$$

$$\langle \psi_{\alpha}^i \bar{\psi}_{\dot{\alpha}}^{\bar{j}} \rangle_{\hat{\varphi}} = i p_{\mu} \sigma_{\alpha\dot{\alpha}}^{\mu} \left[ \left( \hat{K} p^2 - \hat{w} \hat{K}^{-1 T} \hat{w} \right)^{-1} \right]^{i\bar{j}} \quad (30)$$

$$\langle \psi_{\alpha}^i \psi^{\beta j} \rangle_{\hat{\varphi}} = i \delta_{\alpha}^{\beta} \left[ \left( \hat{K} p^2 - \hat{w} \hat{K}^{-1 T} \hat{w} \right)^{-1} \hat{w} \hat{K}^{-1 T} \right]^{ij} \quad (31)$$

From  $\mathcal{L}_{VV}^{(0)}$ , eq. (17), we obtain the  $\hat{\varphi}$ -dressed propagators for the components of quantum vector superfields:

$$\langle A_{\mu}^a A_{\nu}^b \rangle_{\hat{\varphi}} = i \left( -g_{\mu\nu} + \frac{p_{\mu} p_{\nu}}{p^2} \right) \left[ \left( \hat{H} p^2 - \hat{S} \right)^{-1} \right]^{ab} - i \xi \frac{p_{\mu} p_{\nu}}{p^2} \left[ \left( p^2 - \xi \hat{S} \right)^{-1} \right]^{ab} \quad (32)$$

$$\langle G^a \bar{G}^b \rangle_{\hat{\varphi}} = -i \xi \left[ \left( p^2 - \xi \hat{S} \right)^{-1} \right]^{ab} \quad (33)$$

$$\langle D^a D^b \rangle_{\hat{\varphi}} = i p^2 \left[ \left( \hat{H} p^2 - \hat{S} \right)^{-1} \right]^{ab} \quad (34)$$

$$\langle D^a C^b \rangle_{\hat{\varphi}} = -i \left[ \left( \hat{H} p^2 - \hat{S} \right)^{-1} \right]^{ab} \quad (35)$$

$$\langle C^a C^b \rangle_{\hat{\varphi}} = i \frac{1}{p^2} \left[ \left( \hat{H} p^2 - \hat{S} \right)^{-1} \right]^{ab} - i \xi \frac{1}{p^2} \left[ \left( p^2 - \xi \hat{S} \right)^{-1} \right]^{ab} \quad (36)$$

$$\langle \lambda_{\alpha}^a \bar{\lambda}_{\dot{\alpha}}^b \rangle_{\hat{\varphi}} = i p_{\mu} \sigma_{\alpha\dot{\alpha}}^{\mu} \left[ \left( \hat{H} p^2 - \hat{S} \right)^{-1} \right]^{ab} \quad (37)$$

$$\langle \lambda_{\alpha}^a \chi^{\beta b} \rangle_{\hat{\varphi}} = i \delta_{\alpha}^{\beta} \left[ \left( \hat{H} p^2 - \hat{S} \right)^{-1} \right]^{ab} \quad (38)$$

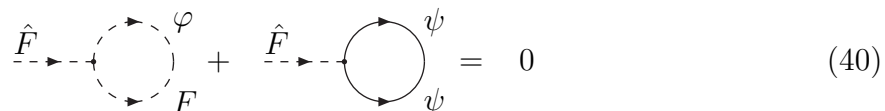
$$\langle \chi_{\alpha}^a \bar{\chi}_{\dot{\alpha}}^b \rangle_{\hat{\varphi}} = i \frac{p_{\mu} \sigma_{\alpha\dot{\alpha}}^{\mu}}{p^2} \left[ \left( \hat{H} p^2 - \hat{S} \right)^{-1} \right]^{ab} - i \xi \frac{p_{\mu} \sigma_{\alpha\dot{\alpha}}^{\mu}}{p^2} \left[ \left( p^2 - \xi \hat{S} \right)^{-1} \right]^{ab} \quad (39)$$



We have not yet taken into account the terms in  $\mathcal{L}_{\phi V}^{(0)}$ , eq. (18), which mix the components of chiral and vector superfields. Such terms could be treated as insertions, to be eventually resummed. This task becomes very easy if supersymmetric Landau gauge is used, that is, if the special value  $\xi = 0$  of the gauge parameter is chosen. It is well known that ordinary Landau gauge simplifies both the computation and the form of the ordinary effective potential [15], because the vector field propagator is transverse and annihilates mixed (scalar-vector) terms. Similarly, supersymmetric Landau gauge simplifies the computation of the effective Kähler potential [3, 5], because the vector superfield propagator is ‘supertransverse’ and annihilates mixed (chiral-vector) terms. So we will stick to this choice<sup>5</sup>. The nice properties of supersymmetric Landau gauge are transferred to the component level, as they should. In particular, the mixed terms in (18) become irrelevant (in the constant  $\hat{\varphi}$  background), so mixed  $\phi V$  propagators are not generated and the  $\phi\phi$  and  $VV$  propagators found above are not modified. Indeed, the combinations of vector superfield components contained in (18) are the same that appear in the gauge fixing lagrangian (last line of (17)), and those components become non-propagating for  $\xi \rightarrow 0$ . As a cross-check, it is easy to verify explicitly that, for  $\xi \rightarrow 0$ , the mixed terms in (18) are annihilated by the vector supermultiplet propagators<sup>6</sup>.

## 5 No corrections to the superpotential

Our next computation is the  $\hat{\varphi}$ -dressed one-point function of  $\hat{F}$ , at one-loop level. This gives us the term  $\Delta w_i(\hat{\varphi})\hat{F}^i$  in the effective lagrangian (15), so we can verify whether or not the superpotential receives a one-loop correction  $\Delta w$ . The quantum bilinears proportional to  $\hat{F}$ , eqs. (21), (22) and (23), should be contracted with propagators, in order to close the  $\hat{F}$  tadpole. In supersymmetric Landau gauge, the required propagators are absent for most of those terms. The only possible contributions to the  $\hat{F}$  tadpole come from the third and fourth terms in (21), which can be closed with propagators. However, the corresponding contributions have equal magnitude and opposite sign, so they cancel each other:



$$\hat{F} \rightarrow \text{dashed circle with } \varphi \text{ and } F + \hat{F} \rightarrow \text{solid circle with } \psi \text{ and } \psi = 0 \quad (40)$$

Thus the total  $\hat{F}$  tadpole vanishes, which we interpret as  $\Delta w = 0$ . For generic  $\xi$ , the same result is obtained as a consequence of more complicated cancellations among several component diagrams involving both chiral and vector supermultiplets (we have checked this on specific examples). The absence of one-loop corrections to the superpotential in the general theory under study can be regarded as an explicit check of the well known non-renormalization theorem [1] (see also [16, 17, 18]). We recall that the literature contains

<sup>5</sup>We will not discuss the  $\xi$  dependence of the one-loop Kähler potential. For such a study in renormalizable theories, where the  $\xi$  dependence affects finite terms, see [5] (and also [4] for the case  $\xi=1$ ).

<sup>6</sup>In more detail:  $FG$  terms do not contribute because now  $G$  has vanishing propagator; mixed terms of the type  $\varphi \partial^\mu A_\mu$  are annihilated by the  $A_\mu$  propagator as usual; similarly, the structure of propagators in the  $(D, C)$  and  $(\lambda, \chi)$  sectors is such that the  $\varphi(D, C)$  and  $\psi(\lambda, \chi)$  terms are annihilated, too.

some examples which violate the theorem at the one- or two-loop level, due to infrared effects associated to massless particles [19, 20, 10]. We do not find such violations, because the background field  $\hat{\varphi}$  generates effective mass terms and thus acts as an effective infrared regulator in field space [15, 17].

## 6 Corrections to the Kähler potential

We now move to the main computation, which is the  $\hat{\varphi}$ -dressed two-point function of  $\hat{F}$ , at one-loop level and vanishing external momenta, in supersymmetric Landau gauge. This will give us the term  $\Delta K_{\bar{i}j}(\hat{\varphi}, \hat{\varphi}) \hat{F}^{\bar{i}} \hat{F}^j$  in the effective lagrangian (15), so we will obtain  $\Delta K$ . The contribution of each diagram to the effective lagrangian is given explicitly, and the integration in (Minkowski) momentum space,  $\int d^4p/(2\pi)^4$ , is denoted by  $\int_p$ . Although the integrals have quadratic or logarithmic ultraviolet divergences, for the time being we do not select a specific regularization. In this respect, notice that no dangerous shifts in the loop momentum  $p$  are needed, since the diagrams are evaluated at vanishing external momenta. Three classes of diagrams have to be considered: they involve the contributions of  $\phi$  multiplets only,  $V$  multiplets only, or both.

*a) Pure  $\phi$  loops.* Using the first interaction term in  $\mathcal{L}_{\phi\phi}^{(\hat{F}\hat{F})}$ , eq. (24), and the first and second ones (+h.c.) in  $\mathcal{L}_{\phi\phi}^{(\hat{F})}$ , eq. (21), we obtain these contributions:

$$\text{Diagram 1} = \hat{F}^{\bar{i}} \hat{F}^j \int_p i \text{Tr} \left[ \hat{K}_{\bar{i}j} (\hat{K} p^2 - \hat{w} \hat{K}^{-1T} \hat{w})^{-1} \right] \quad (41)$$

$$\text{Diagram 2} = -\frac{1}{2} \hat{F}^{\bar{i}} \hat{F}^j \int_p i \text{Tr} \left[ (\hat{K} p^2 - \hat{w} \hat{K}^{-1T} \hat{w})^{-1} \hat{w}_{\bar{i}} (\hat{K}^T p^2 - \hat{w} \hat{K}^{-1} \hat{w})^{-1} \hat{w}_j \right] \quad (42)$$

$$\text{Diagram 3} = -\hat{F}^{\bar{i}} \hat{F}^j \int_p i p^2 \text{Tr} \left[ (\hat{K} p^2 - \hat{w} \hat{K}^{-1T} \hat{w})^{-1} \hat{K}_{\bar{i}} (\hat{K} p^2 - \hat{w} \hat{K}^{-1T} \hat{w})^{-1} \hat{K}_j \right] \quad (43)$$

$$\text{Diagram 4} = -\hat{F}^{\bar{i}} \hat{F}^j \int_p i \text{Tr} \left[ (\hat{K} p^2 - \hat{w} \hat{K}^{-1T} \hat{w})^{-1} \hat{w} \hat{K}^{-1T} \hat{K}_{\bar{i}}^T (\hat{K}^T p^2 - \hat{w} \hat{K}^{-1} \hat{w})^{-1} \hat{w} \hat{K}^{-1} \hat{K}_j \right] \quad (44)$$

$$\text{Diagram 5} = \hat{F}^{\bar{i}} \hat{F}^j \int_p i \text{Tr} \left[ (\hat{K} p^2 - \hat{w} \hat{K}^{-1T} \hat{w})^{-1} \hat{w}_{\bar{i}} (\hat{K}^T p^2 - \hat{w} \hat{K}^{-1} \hat{w})^{-1} \hat{w} \hat{K}^{-1} \hat{K}_j \right] \quad (45)$$

$$\text{Diagram 6} = \hat{F}^{\bar{i}} \hat{F}^j \int_p i \text{Tr} \left[ (\hat{K} p^2 - \hat{w} \hat{K}^{-1T} \hat{w})^{-1} \hat{K}_{\bar{i}} (\hat{K} p^2 - \hat{w} \hat{K}^{-1T} \hat{w})^{-1} \hat{w} \hat{K}^{-1T} \hat{w}_j \right] \quad (46)$$

We recall that here  $\hat{w}$  and  $\hat{K}$  denote the *matrices* defined in (19), and  $\hat{w}_j, \hat{K}_j, \hat{K}_{ij}, \dots$  denote derivatives of those matrices (i.e. third derivatives of  $w$ , third and fourth derivatives of  $K$ ). Two additional diagrams can be built, using the third and fourth terms (+h.c.) in  $\mathcal{L}_{\phi\phi}^{(\hat{F})}$ , eq. (21). However, they cancel each other:

$$\begin{array}{c} \hat{F} \xrightarrow{\varphi} \hat{F} \\ \uparrow \quad \downarrow \\ F \xleftarrow{\varphi} F \end{array} + \begin{array}{c} \hat{F} \xrightarrow{\psi} \hat{F} \\ \uparrow \quad \downarrow \\ \psi \xleftarrow{\psi} \psi \end{array} = 0 \quad (47)$$

*b) Pure  $V$  loops.* Using the interaction terms in  $\mathcal{L}_{VV}^{(FF)}$ , eq. (25), and  $\mathcal{L}_{VV}^{(F)}$ , eq. (22), we obtain:

$$\text{Diagram} = \hat{\overline{F}}^i \hat{F}^j \int_p \frac{i}{p^2} \text{Tr} \left[ \hat{S}_{ij} \left( \hat{H} p^2 - \hat{S} \right)^{-1} \right] \quad (48)$$

$$\text{Diagram: A circle with four external lines. The top and bottom lines are labeled $\hat{F}$, and the left and right lines are labeled $\lambda$. Arrows on the circle indicate a clockwise flow.} = \hat{F}^{\dagger} \hat{F}^j \int_p i p^2 \text{Tr} \left[ \hat{H}_{\bar{i}} (\hat{H} p^2 - \hat{S})^{-1} \hat{H}_j (\hat{H} p^2 - \hat{S})^{-1} \right] \quad (49)$$

$$\begin{array}{c} \chi \\ \text{---}\hat{F}\text{---} \end{array} \begin{array}{c} \text{---}\hat{F}\text{---} \\ \chi \end{array} = \hat{F}^{\dagger} \hat{F}^j \int_p \frac{i}{p^2} \text{Tr} \left[ \hat{S}_i (\hat{H} p^2 - \hat{S})^{-1} \hat{S}_j (\hat{H} p^2 - \hat{S})^{-1} \right] \quad (50)$$

$$\text{Diagram: A circle with four external lines. Top-left: incoming line with label } \hat{F} \text{ and arrow pointing in. Top-right: outgoing line with label } \hat{F} \text{ and arrow pointing out. Bottom-left: incoming line with label } \lambda \text{ and arrow pointing in. Bottom-right: outgoing line with label } \chi \text{ and arrow pointing out. Internal lines: Top arc labeled } \chi \text{ with arrow pointing clockwise. Bottom arc labeled } \lambda \text{ with arrow pointing clockwise.} = -\hat{F}^{\bar{i}} \hat{F}^j \int_p i \text{Tr} \left[ \hat{S}_{\bar{\tau}} (\hat{H} p^2 - \hat{S})^{-1} \hat{H}_j (\hat{H} p^2 - \hat{S})^{-1} \right] \quad (51)$$

$$\begin{array}{c} \chi \\ \text{---}\hat{F}\text{---} \end{array} \begin{array}{c} \text{---}\hat{F}\text{---} \\ \chi \end{array} = -\hat{F}^{\bar{i}}\hat{F}^j \int_p i \text{Tr} \left[ \hat{H}_{\bar{i}} \left( \hat{H} p^2 - \hat{S} \right)^{-1} \hat{S}_j \left( \hat{H} p^2 - \hat{S} \right)^{-1} \right] \quad (52)$$

In the above expressions,  $\hat{S}_j, \hat{H}_j, \hat{H}_{\bar{j}j}, \dots$  denote derivatives of the matrices  $\hat{S}$  and  $\hat{H}$  defined in (20). Also, the trace operation does not include the trace in spinor space: the latter has already been performed.

c) *Mixed  $\phi$ - $V$  loops.* Two non-vanishing diagrams can be built using the interaction terms in  $\mathcal{L}_{\phi V}^{(\hat{F})}$ , eq. (23), but they cancel each other:

$$\begin{array}{c} \hat{F} \end{array} \begin{array}{c} \xrightarrow{F} \\ \xrightarrow{F} \\ \xrightarrow{F} \end{array} \begin{array}{c} \hat{F} \end{array} + \begin{array}{c} \hat{F} \end{array} \begin{array}{c} \xrightarrow{\psi} \\ \xrightarrow{\psi} \\ \xrightarrow{\psi} \end{array} \begin{array}{c} \hat{F} \end{array} = 0 \quad (53)$$

This cancellation is a further effect of supersymmetric Landau gauge at the component level. In superspace, the vanishing of mixed  $\phi$ - $V$  loops should automatically follow from mixed  $\phi$ - $V$  vertices being annihilated by the supertransverse  $V$  propagator, as in the renormalizable case [5].

Now we have to sum the diagrams above and express the coefficient of  $\hat{\bar{F}}^i \hat{F}^j$  as a second derivative (with respect to  $\hat{\bar{\varphi}}^i$  and  $\hat{\varphi}^j$ ). After some manipulations, the results for the  $\phi$  and  $V$  sectors can be cast in the required form:

$$\hat{\bar{F}}^i \rightarrow \text{---} \bigcirc \phi \text{---} \hat{F}^j = \hat{\bar{F}}^i \hat{F}^j \int_p \frac{i}{p^2} \left[ \frac{1}{2} \text{Tr} \log \hat{K} + \frac{1}{2} \text{Tr} \log \left( \hat{K} p^2 - \hat{w} \hat{K}^{-1 T} \hat{w} \right) \right]_{\bar{i}j} \quad (54)$$

$$\hat{\bar{F}}^i \rightarrow \text{---} \bigcirc V \text{---} \hat{F}^j = \hat{\bar{F}}^i \hat{F}^j \int_p \frac{i}{p^2} \left[ -\text{Tr} \log \left( \hat{H} p^2 - \hat{S} \right) \right]_{\bar{i}j} \quad (55)$$

Finally, from the comparison of eqs. (54) and (55) with eq. (15), we can read off the one-loop correction to the Kähler potential:

$$\Delta K(\hat{\bar{\varphi}}, \hat{\varphi}) = \int_p \frac{i}{p^2} \left[ \frac{1}{2} \text{Tr} \log \hat{K} + \frac{1}{2} \text{Tr} \log \left( \hat{K} p^2 - \hat{w} \hat{K}^{-1 T} \hat{w} \right) - \text{Tr} \log \left( \hat{H} p^2 - \hat{S} \right) \right]. \quad (56)$$

We can go one step further and perform the momentum integration. If we do a Wick rotation and regulate the momentum integral with a simple ultraviolet cutoff  $\Lambda$ , the result reads:

$$\begin{aligned} \Delta K(\hat{\bar{\varphi}}, \hat{\varphi}) &= \frac{\Lambda^2}{16\pi^2} \left[ \log \det \hat{K} - \log \det \hat{H} \right] \\ &- \frac{1}{32\pi^2} \left[ \text{Tr} \left( \mathcal{M}_\phi^2 \left( \log \frac{\mathcal{M}_\phi^2}{\Lambda^2} - 1 \right) \right) - 2 \text{Tr} \left( \mathcal{M}_V^2 \left( \log \frac{\mathcal{M}_V^2}{\Lambda^2} - 1 \right) \right) \right] \end{aligned} \quad (57)$$

where  $\mathcal{M}_\phi^2$  and  $\mathcal{M}_V^2$  are field dependent mass matrices in the chiral and vector sectors:

$$\mathcal{M}_\phi^2 \equiv \hat{K}^{-1/2} \hat{w} \hat{K}^{-1 T} \hat{w} \hat{K}^{-1/2}, \quad \mathcal{M}_V^2 \equiv \hat{H}^{-1/2} \hat{S} \hat{H}^{-1/2}. \quad (58)$$

We recall that, in the above expressions, all the dependence on  $(\hat{\bar{\varphi}}, \hat{\varphi})$  is contained in the matrices  $\hat{K}, \hat{w}, \hat{H}, \hat{S}$  defined in (19) and (20). The functional dependence of  $\Delta K$  is what we were looking for. Indeed, if we recall that eq. (15) was derived from eq. (10), it is clear that we can go back to superspace and replace the arguments  $(\hat{\bar{\varphi}}, \hat{\varphi})$  of  $\Delta K$  with general superfields  $(\hat{\bar{\phi}}, \hat{\phi})$ , or even with  $(\hat{\bar{\phi}}, e^{2\hat{V}} \hat{\phi})$  in the case of a background gauge invariant quantization. So eq. (57) is our final result: it gives the full one-loop correction to the Kähler potential in a closed form, for the general theory under study. The first line of eq. (57) contains quadratically divergent contributions, whereas the second line contains logarithmically divergent and finite contributions. If we evaluate the momentum integral in  $d=4-2\epsilon$  dimensions ( $\int_p = \mu^{2\epsilon} \int d^d p / (2\pi)^d$ ) instead of using a momentum cutoff in  $d=4$ , the first line of eq. (57) should be omitted, and the replacement  $\log \Lambda^2 \rightarrow 1/\epsilon + 1 - \gamma + \log(4\pi\mu^2)$  should be made in the second line<sup>7</sup>.

In the special case of renormalizable theories, the superpotential is at most cubic and the metric is canonical in both the chiral and vector sectors, so  $\hat{K}_{\bar{i}j} = \delta_{\bar{i}j} = \text{const.}$ ,

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<sup>7</sup> Notice that this regularization corresponds to dimensional reduction [21], since the spinor algebra has been performed in  $d=4$ .

$\hat{w}_{ij} = m_{ij} + h_{ijk}\hat{\varphi}^k$ ,  $\hat{H}_{ab} = \delta_{ab}/g_a^2 = \text{const.}$ ,  $\hat{S}_{ab} = \hat{\varphi}\{T_a, T_b\}\hat{\varphi}$ . In this limit, the first line of (57) becomes irrelevant and the second line reproduces the result derived in [5], if  $g_a^2$  is identified with  $2g^2$ . Incidentally, we recall that the result of [5] was obtained by computing two superdiagrams, i.e. one in each sector, after resumming  $\hat{\varphi}$  insertions. Our component field approach is not much more involved. Using  $\hat{\varphi}$ -dressed propagators amounts to resumming  $\hat{\varphi}$  insertions, and our result for  $\Delta K$  originates from only three  $\hat{F}\hat{F}$  component diagrams in the renormalizable case, i.e. (42) in the chiral sector and (48), (50) in the vector sector.

In the case of non-renormalizable theories, the quadratically divergent contributions in the first line of (57) agree with the results of ref. [6], obtained with superfield methods. In our approach, those contributions can be traced back to the component diagrams (41), (43) and (49). We have also tried to make a comparison with ref. [7], in which detailed computations of the one-loop bosonic effective action in general supergravity theories with diagonal gauge kinetic function were presented, and the flat limit was also considered. Only the divergent contributions were evaluated, and part of them was interpreted as a correction to the Kähler potential. A component field approach different from ours was used. Here we have insisted on preserving supersymmetry and supersymmetric background gauge invariance. In [7] the Wess-Zumino gauge was used, and special emphasis was given to ordinary background gauge invariance and scalar field reparametrization covariance. We recall that the Wess-Zumino gauge generally leads to a loss of manifest supersymmetry, since vector supermultiplets are integrated out in a non-supersymmetric way. To compensate for this, a special  $R_\xi$ -type gauge fixing for the component vector fields was introduced in [7], with  $\xi=1$ . This particular prescription was argued to restore supersymmetry, since the anomalous dimensions of component scalar fields were found to coincide with the supersymmetric ones of the associated chiral superfields, in the flat limit. Although this coincidence may be partly accidental<sup>8</sup>, the divergent part of the one-loop Kähler potential reconstructed in [7] seems to agree with ours. Strictly speaking, a slight difference can be found, for another reason. Indeed, the derivatives in  $\hat{w}_{ij}$  are reparametrization covariant ones in the formulae of [7] (see also [8]). However, this apparent discrepancy is not a physical one: it depends on the way the background-quantum splitting of chiral supermultiplets is performed<sup>9</sup>. If desired, our result could be made reparametrization covariant *a posteriori*, e.g. by reinterpreting the derivatives in  $\hat{w}_{ij}$  as covariant ones and promoting  $(T_a\hat{\varphi})^i$  to a more general holomorphic Killing vector  $v_a^i(\hat{\varphi})$ . This could perhaps be confirmed by a supersymmetric normal coordinate expansion [23, 6], which however goes beyond the scope of the present paper.

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<sup>8</sup>For instance, extending that coincidence to the fermionic components of chiral superfields would require some additional modification. As a further example of one-loop computations in the Wess-Zumino gauge, we may recall the component approach employed in [22] to study the divergences in a general renormalizable theory. In this case ordinary Landau gauge was used for the component vector fields, and the scalar field anomalous dimensions did not coincide with the chiral superfield ones. The latter were reconstructed from the renormalization of superpotential parameters.

<sup>9</sup>We recall that the perturbative computations and the resulting effective action depend on both that choice and other ones, such as the background-quantum splitting of vector supermultiplets and the choice of the gauge fixing function (or parameter). All such ambiguities are expected to disappear at the level of the physical S-matrix.

## 7 A consistency check

We conclude by presenting an alternative derivation of  $\Delta K$ . This derivation is based on background gauge invariance, and the agreement of the final result with the one found above provides an interesting consistency check. We recall that, although we have presented our general framework in a background gauge invariant way, the latter property has not been exploited in the previous section, where the functional form of  $\Delta K$  has been computed by using a background with vanishing  $\hat{V}$ . Strictly speaking, what we obtained was  $\Delta K(\hat{\tilde{\phi}}, \hat{\phi})$ . If the background-quantum splitting of vector superfields and the gauge fixing term are chosen as in (7) and (8), the result should be automatically promoted to  $\Delta K(\hat{\tilde{\phi}}, e^{2\hat{V}}\hat{\phi})$ . We want to check this explicitly, so we switch on a non-vanishing  $\hat{V}$ , which we take to consist of a constant  $\hat{D}$  field, as in eq. (14). At the same time, we take the background superfield  $\hat{\phi}$  in the form (13), with constant  $\hat{\varphi}$  but vanishing  $\hat{F}$ . Then we compute the terms linear in  $\hat{D}$  in the one-loop effective lagrangian (or potential), which eq. (15) predicts to have the form  $\hat{D}^a[\Delta K_j(\hat{\tilde{\varphi}}, \hat{\varphi})(T_a\hat{\varphi})^j + \Delta\kappa_a]$ . Thus  $\Delta K$  can be reconstructed and compared to the previous result, and  $\Delta\kappa_a$  can be identified as well.

In order to check all this, we first have to expand the lagrangian in the new background and find the terms bilinear in quantum fields ( $\mathcal{L}_{\text{bil}}$ ) that have a linear dependence on  $\hat{D}$ . We omit the full list because only a few among such terms give a non-vanishing  $\hat{D}$  tadpole, in supersymmetric Landau gauge. For instance, terms that couple  $\hat{D}$  to a mixed  $\phi$ - $V$  bilinear cannot contribute, due to the absence of mixed propagators. Also, terms that couple  $\hat{D}$  to ghost bilinears are  $\hat{\varphi}$  independent and could at most contribute to the Fayet-Iliopoulos term, but the actual contribution is zero because ghosts belong to the adjoint representation, which is vector-like. Special care is needed to study the effect of the gauge fixing lagrangian (8), because it generates terms that couple  $\hat{D}^a$  to the components of quantum supermultiplets  $V^b, V^c$  with strength  $\sim c_{bc}^a/\xi$ . Since this coefficient is divergent in the limit  $\xi \rightarrow 0$ , a small non-vanishing  $\xi$  should be kept in intermediate steps, and the mixed  $\phi$ - $V$  terms in  $\mathcal{L}_{\phi V}^{(0)}$ , eq. (18), should be taken into account. When pure  $V^b$ - $V^c$  propagators are used to close the  $\hat{D}$  tadpole, the integrand is zero because the propagators are  $bc$ -symmetric whereas the structure constants are antisymmetric<sup>10</sup>. If mixed  $\phi$ - $V$  insertions are used, at least two of them are needed to close the loop. However, since they pick up only the  $\xi$  dependent parts of the adjacent  $V$  propagators, the singular  $1/\xi$  factor is multiplied by a factor at least  $\mathcal{O}(\xi^2)$ , so the result is again zero in the limit  $\xi \rightarrow 0$ . After completing the inspection of these and other terms, we find that the only terms that can give a non-vanishing  $\hat{D}$  tadpole are<sup>11</sup>:

$$\mathcal{L}_{\phi\phi}^{(\hat{D})} = \hat{D}^a \left( \hat{K}_{\tilde{t}k}(T_a)^k_j + \hat{K}_{\tilde{t}kj}(T_a\hat{\varphi})^k \right) \bar{\varphi}^{\tilde{t}} \varphi^j \quad (59)$$

$$\mathcal{L}_{VV}^{(\hat{D})} = \hat{D}^a \left[ \frac{1}{2}(\hat{S}_{bc})_j(T_a\hat{\varphi})^j C^b C^c + \frac{i}{2}\hat{f}_{bd}c_{ac}^d(D^b C^c - \lambda^b \chi^c) \right] + \text{h.c.} \quad (60)$$

The contributions to the  $\hat{\varphi}$ -dressed  $\hat{D}$  tadpole are, in matrix notation:

<sup>10</sup> A similar mechanism also kills other terms in the lagrangian.

<sup>11</sup>We remark that the background-quantum splitting of the vector superfield, eq. (7), plays a crucial role here. The simple splitting  $\hat{V} \rightarrow \hat{V} + V$  would not give the desired result, for a non-abelian gauge group.



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